Vol.X, 3,4 BOLETIN DE LA SOCIEDAD MATEMATICA MEXICANA 1953

THE INTRODUCTION TO THE THEORY OF ANALYTIC FUNCTIONS*

Marshall H. Stone.

A pedagogical very satisfactory approach to the study of integration originates in the search for the primitives of a given function - that is, for those functions having the given one as derivative. In the case of real functions of a real variable this approach has traditional advantages which, as we point out here, are preserved in the consideration of complex functions of a complex variable. The study of the equation

$$\frac{dw}{dz} = f(z) \tag{1}$$

where f is a given continuos function of the complex variable z with domain D, lead directly and simply to the essential

*Received September, 1951.

results of the Cauchy theory. The technical apparatus required in this development is at a minimum. In the standard theory all the preliminar labor needed to define integration along a path is promply shown by the Cauchy integral theorem to have been essentialy irrelevant; but here contour integration is a relegated to its proper place as a useful devise for computation. Any solution of (1) has an increment along the directed segment z_1, z_2 which is given by the integration of f as a function of a *real* parameter for the segment. This integral exists in any case, since f is continuos; and its value defines a function $H(z_1, z_2)$ of *two* complex variables on any convex part C of D. A solution of (1) will exists if and only if $H(z_1, z_2)$ is expressible as the increment of a function of one variable; or, equivalently, if

$$(2) \qquad H(z_1, z_2) + H(z_2, z_3) + H(z_3, z_1) = 0 \quad .$$

A *sufficient* condition for (2) to hold is that f have a derivative, the proof proceeds, along traditional and here clearly motivated lines, by subdivision of the triangle with vertices at z_1, z_2, z_3. The problem of matching local solutions of (1) to obtain a global one presents itself here as an elementary problem of combinatorial topology. Its solution for any simply connected open part of D contains the essence of the Cauchy integral theorem and leads at once to this theorem and the associated Cauchy integral formula. A familiar argument then shows that a function wich has a derivative everywhere in an open set is infinitely differentiable there, and is expressible locally by mean of power series. The *sufficient* condition above then appears to be *necessary* as well.

Chicago University.